PG – 387

III Semester M.Sc. Degree Examination, December 2016
(CBCS)
MATHEMATICS
M 301T : Differential Geometry

Time : 3 Hours

Max. Marks : 70

Instructions : 1) *Answer **any five** questions.*
2) ***All** questions carry **equal** marks.*

1. a) Define directional derivative of a differentiable function by a tangent vector. If $V_p = (v_1, v_2, v_3)_p$ is a tangent vector to E^3 at p and f is a real valued differentiable function on E^3 then show that the directional derivative $V_p[f] = \sum_{i=1}^{3} v_i \frac{\partial f}{\partial x_i}(p)$. Use it to prove $U_i[f] = \frac{\partial f}{\partial x_i}$ for a natural frame field (U_1, U_2, U_3) on E^3. 7
   b) If $V = x U_1 - y^2 U_3$, $f = x^2 y + z^3$, and $g = xy$ then compute V[f], V[g] and V[fg]. 7

2. a) Define reparametrisation of a curve. Show that every regular curve has a unit speed reparametrisation. 6
   b) Reparametrise the curve $\alpha(t) = (a\cos t, a\sin t, bt)$ by its arc length, where $b > 0$. 5
   c) Evaluate the 1-form $\phi = x^2 dx - y^2 dz$ on the vector field $V = xU_1 + yU_2 + zU_3$. 3

3. a) If ϕ and ψ are any two 1-forms on E^3 then prove that $d(\phi \wedge \psi) = (d\phi \wedge \psi - \phi \wedge d\psi)$. 6
   b) If β is a unit speed curve with constant curvature $k > 0$ and torsion zero then prove that β is part of a circle of radius $1/k$. 4
   c) Show that the curve $\beta(s) = \left(\frac{4}{5}\cos s, 1 - \sin s, \frac{3}{5}\cos s\right)$ is a circle. 4

4. a) Let $V = (1, -1, 2)$, $P = (1, 3, -1)$ and $W = x^2 U_1 + yU_2$. Then compute $\nabla_{V_p} W$. 4
   b) Obtain the connection forms for a cylindrical frame field. 5
   c) Prove that a translation, a rotation and an orthogonal transformation are isometries. 5

P.T.O.

5. a) Define a proper patch. If f is a real valued differentiable function on E^3 then prove that the map $X : D \subset E^2 \to E^3$; $X(u, v) = (u, v, f(u, v))$ $\forall (u, v) \in D$ is a proper patch. 6

b) Is cylinder in E^3, a surface ? Justify. 4

c) If $M : g(x, y, z) = C$ is a surface in E^3 then show that the gradient vector field $\nabla g = \sum \frac{\partial g}{\partial x_i} U_i$ is a non vanishing normal vector field on E^3. 4

6. a) Obtain parametrisation of the following :

i) A cylinder in E^3.

ii) Entire surface obtained by revolving the curve $C : y = \cosh x$ around x – axis. 6

b) With usual notations prove

i) $d(F^*\xi) = F^*(d\xi)$, for any 1 – form ξ

ii) $X^*(\phi) = \phi(X_u)du + \phi(X_v)dv$ for any 1 – form ϕ.

iii) $X^*(d\phi) = \left(\frac{\partial}{\partial u}(\phi(X_v)) - \frac{\partial}{\partial v}(\phi(X_u))\right)du\,dv$. 8

7. a) Define shape operator of a surface at a point. Show that it is a linear operator. 5

b) With usual notations prove $K = k_1 k_2$, $H = \frac{1}{2}(k_1 + k_2)$. 4

c) If V and W are linearly independent tangent vectors at a point P of $M \subset E^3$ then prove that

i) $S(V) \times S(W) = K(P)\, V \times W$.

ii) $SV \times W + V \times SW = 2\,H(P)\, V \times W$. 5

8. a) Compute the Gaussian, the mean curvatures and hence the principal curvatures k_1, k_2 for the surface $X(u, v) = (u \cos v, u \sin v, bv)$, $b \neq 0$. 4

b) Let α be a regular curve in a surface M in E^3 and let U be a unit normal vector field restricted to α. Then prove that the curve α is principal if and only if U' and α' are collinear at each point. 6

c) Determine the geodesics in

i) a plane ii) a sphere 4